

Set Theory, pt.3 (Solution Sets)

An open sentence is an equation or inequality with one or more variables.
(such as $x + 5 = 7$, or $y > 3$).

A solution set is the set whose members contain all the true solutions for an open sentence.

Ex. $\{2\}$ is the sol'n set for $x + 5 = 7$, and $(3, 4)$ is the sol'n set for $y > 3$.

- $(3, 4)$ is the open interval between 3 and infinity. It represents all the real numbers between (but not including) 3 and infinity.

Each member of a solution set is said to be a solution for the open sentence.

A solution of an equation is called a root of the equation.

Ex. Find all real roots (solutions) for the equation $x^2 = 4$.

Ans. Sol'n set = $\{-2, 2\}$ since both $-2^2 = 4$ and $2^2 = 4$.

$x = -2, 2$ are the roots of the equation.

An open interval is represented by (a, b) where a and b are two real numbers and (a, b) represents all real numbers between but not including a and b .

A closed interval is represented by $[a, b]$ where a and b are two real numbers and $[a, b]$ represents all real numbers between and including a and b .

$[a, b)$ is an interval that contains all real numbers between a and b , and the real number a , but not the real number b .

$(a, b]$ is an interval that contains all real numbers between a and b , and the real number b , but not the real number a .

* When $a = -\infty$, or $b = \infty$ an open interval is used, since infinity is not an actual number but rather a representation of a continuing, non-ending nature.

Ex. the sol'n set for $2 \leq x < 3$ is $[-2, 3)$

Ex. the sol'n set for $-3 \leq x < 3$ is $[-3, 3)$

Exercises: find the solution sets for each of the following:

- 1.) $a > 5$
- 2.) $a + 3 > 5$
- 3.) $a - 2 \leq 5$
- 4.) $9 < x < 15$
- 5.) $9 \leq x < 15$
- 6.) $9 < x \leq 15$
- 7.) $9 < x - 2 \leq 15$
- 8.) $9 \leq 2x + 3 < 15$

To solve such an inequality as #7 and #8, separate into two inequalities, and find the intersection of the sol'n sets.

Find the roots of the following equations: (solve for the variable)

- 9.) $x^2 = 9$

$$10.) x + 5 = 7$$