Properties of Powers

If we take a number and multiply it by itself, we call the resultant product a square of the number. We can represent the product as the 1rst number with a 2nd number in superscript (called the exponent), meaning the number of times a number (called the base) is multiplied by itself.

Ex.

$3^2 = 3 \text{ x } 3 = 9$

$a^2 = a \text{ x } a$

If we desire to add powers, they must have the same base and exponent..

Ex. $x^2 + 3x^2 + x^3 = x^3 + 4x^2$

*Note: unlike powers will not combine.

Ex. $3x^2y + x^2 = 3x^2y + x^2 = (3y+1)x^2$ Distributive rule of equality

Now,

If we want to multiply two powers; we need to check to see if the bases are the same or different.

1.) If the bases are different, it looks like this;

$X * y = xy$; $x^2 * y = x^2y$

2.)If the bases are the same, then you can add the exponents to find the new power.

Ex.

$x^2 * x^3 = x^{2+3} = x^5$

Ex.

$2^2 * 2^3 = (2*2)(2*2*2) = 2*2*2*2*2 = 2^5$

so for any base times itself

$$x^a * x^b = x^{a+b}$$

What if we raise a power to a power? Remember that an exponent means the base is multiplied by itself that many times. So:

$(x^2)^3 = x^2 * x2 * x^2 = x^6$

so in general,

$$(x^a)b = x^{a*b}$$

If we divide powers we again have two possibilities.

1.) If the bases are different, we can leave it as a fraction (or we can rewrite the denominator as a negative exponent)

Ex. $x^2 \div y^3 = \dfrac{x^2}{y^3} = x^2 y^{-3}$

2.) If the bases are the same, then we can reduce to a single power whose exponent is the difference of the numerator and the denominator.

$$x^5 \div x^2 = \frac{x^5}{x^2} = \frac{\not{x} \cdot \not{x} \cdot x \cdot x \cdot x}{\not{x} \cdot \not{x}} = x^3$$

or in general,

$$\frac{x^a}{x^b} = x^{a-b}$$

notice: that a number in the denominator can be written as a negative exponent. We can use this to write powers in the denominator . In other words:

$$\frac{1}{x^a} = x^{-a}$$

So what if we have the same power in the numerator and denominator? The equivalent power would be the base to 0. But the same number divided by itself equals one.

So, $\frac{x^a}{x^a} = 1$ and $\frac{x^a}{x^a} = x^{a-a} = x^0$ $\therefore \boxed{x^0 = 1}$

What if we have a base raised to 1? Then by definition we have only one base to multiply, so by definition:

$$\boxed{x^1 = x}$$

What if the exponent is a fraction? Looking above at the definition of x1 = x and using the multiplication of powers, we look at the following example:

$$x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^1 = x$$

since $x^{\frac{1}{2}} = x^{\frac{1}{2}}$ we can write

$$(x^{\frac{1}{2}})^2 = x^1 \text{ or } x^{\frac{1}{2} \cdot 2} = x$$

since the number that multiplied by itself will give us x, the number is the square root of x ($\sqrt{x}$),

we can say that

$$\boxed{x^{\frac{1}{2}} = \sqrt{x}}$$

or in general

$$\boxed{x^{\frac{1}{a}} = \sqrt[a]{x}}$$

and

$$\boxed{x^{\frac{b}{a}} = \sqrt[a]{x^b}}$$

notice that in the above if a = b, then:

$$x^{\frac{b}{a}} = \sqrt[a]{x^b} = (\sqrt[a]{x})^b = \sqrt[a]{x^a} = (\sqrt[a]{x})^a = x$$